UNIT -1

NUMBERS

Place Value or Local Value Of a Digit in a Numeral

In the number 68974532, we have:

Place value of 2 is 2 units = 2;

Place value of 3 is 3 tens = 30;

Place value of 5 is 5 hundreds = 500;

Place value of 4 is 4 thousands = 4000 and so on.

Face Value: The face value of a digit in a numeral is the value of digit itself at whatever place it may be.

In the above numeral, the face value of 2 is 2; the face value of 3 is 3 and so on.

Natural Numbers: Counting numbers 1,2,3,4,5…. Are known as natural numbers.

Even Numbers: A number divisible by 2 is called an even number

e.g. 2,4,6,8.,10 etc.

Odd Numbers: A number not divisible by 2 is called an odd number

e.g. 1,3,5,7,9,11,13,15,17 etc.

Ex. Find the unit's digit in the product (256*27*159*182)

Sol. Product of unit's digit in given numbers = (6*7*9*2) = 756

.'. Unit digit in the given product is 6.

TESTS OF DIVISIBLITY:

Divisibility by 2: A number is divisible by 2 if its unit digit is any of 0,2,4,6,8.

Divisibility by 3: A number is divisible by 2 if the sum of its digits is divisible by 3.

Divisibility by 9: A number is divisible by 9, if the sum of its digits is divisible by 9

Divisibility by 4: A number is divisible by 4, if the number formed by the last two digits divisible by 4.

Divisibility by 8: A number is divisible by 8, if the number formed by the last 3 digits of the given number is divisible by 8.

Divisibility by 11: A number is divisible by 11, if the difference of the sum of its digits at odd places and the sum of its digits at even places, is either 0 or a number divisible by 11.

Ex. Which of the following numbers is not divisible by 2?

3567516, 9130525, 7870832 & 1371594

Sol. Clearly, the unit's digit of 9130525 is 5, which is not divisible by 2.

So, 9130525 is not divisible by 2.

Each one of the other numbers has a unit digit divisible by 2.

So, each one other then 9130525 is divisible by 2.

Ex. Which of the following numbers is not divisible by 8?

(i) 98016542 (ii) 106598304

Sol. (i)The number formed by the last 3 digits of the given number is 542.

Which is not divisible by 8.

.'. 98016542 is not divisible by 8.

(ii) The number formed by the last 3 digits of the given number 304, which is divisible by 8

.'. 106598304 s divisible by 8.

Ex. Find the sum of all odd numbers upto 100.

Sol. The given numbers are 1,3,5,7…..,99.

This is an A.P. with a= 1 and d=2.

Let it contains n terms. Then,

1 + (n – 1) * 2 = 99 or n = 50.

.'. Required sum = $\frac{n}{2}$(first term + last term).

= $\frac{50}{2}$ * (1 + 99) = 2500.

H.C.F and L.C.M OF NUMBERS

Factors and multiples: If a number a divides another number b exactly, we say that a is a factor of b and we write a/b. In this case, b is called a multiple of a.

Highest common factor or Greatest common measures:

The H.C.F of two or more than two numbers is the greatest number that divides one of them exactly.

H.C.F By Factorisation: Express each one of the given numbers as the product of prime factors. The product of least powers of common prime factors gives H.C.F.

H.C.F By Division Method: Suppose we have to find the H.C.F of two given numbers. Divide the larger number by the smaller one. Now , divide the divisor by the remainder. Repeat the process of dividing the preceding divisor by the remainder last obtained till zero is obtained as remainder. The last divisor is the required H.C.F.

Suppose we have to find the H.C.F of three numbers. Then, H.C.F of (HCF of any two and the third number) gives the H.C.F. of three given numbers. Similarly, the HCF of more than three numbers may be obtained.

Ex.1.What is the H.C.F of $2^3x3^2x5x7^4$, $2^2x3^3x5^2x7^3$,$2^3x5^3x7^2$?

Sol. The prime numbers common to given numbers are 2 , 5 and 7.

H.C.F = 2^2X5X7^2 = 980.

Ex.2. Find the H.C.F of 108, 288 and 360.

Sol. 108 = 2^2x3^3, 288 = 2^5x3^2 and 360 = 2^3x5x3^2.

H.C.F = 2^2X3^2 = 36.

Ex.3. Find the H.C.F of 1056 , 1584 and 2178.

$x3^2x11^2$.

Sol. 1056 = 2^5x3x11, 1584 = 2^4x3^2x11 and 2178 = 2

H.C.F = 2X 3X 11 = 66.

LEAST COMMON MULTIPLE(L.C.M): The least number which is exactly divisible by each one of the given numbers is called their L.C.M.

Product of Two Numbers=Product of their HCF and LCM.

L.C.M By Factorisation: Resolve each one of the given numbersinto a product of prime factors. Then , L.C.M is the product of highest powers of all the factors.

Ex.1. Find the L.C.M of $2^2x3^3x5x7^2$, $2^3x3^2x5^2x7^4$, $2x3x5^3x7x11$.

Sol. LCM = Product of highest powers of 2,3,5,7 and 11.

$=2^{3x}3^3x5^3x7^4x11$.

Ex.2. Find the LCM of 72, 108 , and 2100.

Sol. $72 = 2^3x3^2$, $108= 3^3x2^2$, $2100 = 2^2x5^2x3x7$.

L.C.M = $2^3X3^3X5^2X7= 37800$

Ex.3. Find L.C.M of 852 and 1491.

Sol. H.C.F of 852 and 1491 is 213.

L.C.M = Product of numbers/Their H.C.F

$=(852 X 1491)/213 = 5964$.

SIMPLIFICATION

In simplifying an expression, first of all bar must be removed. After removing the bar, the brackets must be removed , strictly in order (),{} and[].

After removing the brackets, we must use the following operations strictly in the order:

(i) Of

(ii) Division

(iii) Multiplication

(iv) Addition

(v) Subtraction

REMARK: Remember the word `BODMAS` where B,O,D,M,A, and S stand for bracket, of, division, multiplication, addition and subtraction respectively.

Ex. 1. Simplify : $108 \div 36 \text{ of } \frac{1}{4} + \frac{2}{5} + 3\frac{1}{4}$.

Sol. Given expression = $108 \div 9 + \frac{2}{5} * \frac{13}{4}$

$= \frac{108}{9} + \frac{13}{4} = (12 + \frac{13}{10}) = \frac{133}{10} = 13\frac{3}{10}$.

Ex .2. Simplify : $\frac{1}{3} \div \frac{5}{3} + \frac{1}{4} * \frac{3}{5} - \frac{2}{5} of \frac{5}{7}$.

Sol. given expression $= \frac{1}{3} \div \frac{5}{3} + \frac{1}{4} * \frac{3}{5} - \frac{2}{5} of \frac{5}{7}$

$= \frac{1}{3} \div \frac{5}{3} + \frac{1}{4} * \frac{3}{5} - \frac{2}{7}$

$= \frac{1}{3} * \frac{3}{5} + \frac{3}{20} - \frac{2}{7}$

$= \frac{1}{5} + \frac{3}{20} - \frac{2}{7}$

$= \frac{4+3}{20} - \frac{2}{7} = \frac{7}{20} - \frac{2}{7}$

$= (\frac{49-40}{140}) = \frac{9}{140}$.

Ex. 3. Simplify :3.5÷ 0.7of 7+0.5*0.3-0.1

Sol. given expression =3.5÷ 0.7+7+0.5*0.3-0.1

=3.5÷ 4.9+0.5*0.3-0.1

$=\frac{3.5}{4.9}+0.15-0.1$ $=\frac{5}{7}+\frac{15}{100}-\frac{1}{10}$

$=\left(\frac{100+7}{140}\right)=0.764$

Ex. 4. Simplify : 2-[3-{6-{(5-$\overline{4-3}$)}]

Sol. given expression = 2-[3-{6-(5-1)}]

=2-[3-{6-4}] = 2-[3-2]=(2-1)=1.

Ex. 5. A boy was asked to multiply a certain number by 53. He multiplied it by 35 and got his answer less than the correct one by 1206. Find the number to be multiplied.

Sol. let the required no be x

Then 53x-35x=1206

Then x =67 is equal to required number.

UNIT - 2

TIME AND WORK

GENERAL CONCEPTS:

- If A can do a piece of work in n days, then A's 1 day's work= 1/n
- If A's 1 day's work = 1/n then A can finish the work in n days.
- If A is thrice as good as B then:
    - Ratio of work done by A and B =3:1
    - Ratio of times taken by A and B to finish a work = 1:3.

Ex.1 . A can do a piece of work in 10 days which B alone can do in 12 days. In how many days will they finish the work, both working together ?

Sol: A's 1 day's work=1/10, B's 1 day's work=1/12.

(A+B)'s 1 day's work=$\left(\frac{1}{10}+\frac{1}{12}\right)=\frac{11.}{60}$

$\therefore$ both will finish the work in 60/11 days.

Ex.2. Two persons A and B working together can dig a trench in 8 hours while A alone can dig it in 12 hours. In how much hours B alone can dig the trench?

Sol: (A+B)'s 1 hour's work= 1/8

A's 1 hour work=1/12

$\therefore$ B's 1 hour work =$\left(\frac{1}{8}-\frac{1}{12}\right)=\frac{1.}{24}$

Hence B alone can dig it in 24 hours.

EX.3. A is twice as good a workman as B and together they finish a piece of work in 18 days. In how many days will A alone finish he work?

Sol: (A's 1 days work): (B's 1 days work)=2:1

(A+B)'s 1 days work=1/18.

Divide 1/18 in the ratio 2:1.

So, A's 1 days work=$\left(\frac{1}{18}\times\frac{2}{3}\right)=\frac{1}{27}$

Hence A alone can finish the work in 27 days.

TIME AND DISTANCE

FORMULAE:

i) speed=$\left(\frac{distance}{time}\right)$, time=$\left(\frac{distance}{speed}\right)$

ii) distance = speed x time

iii) 1km/hr=$\frac{\frac{5}{18}m}{\text{sec}}$,1m/sec=$\frac{\frac{18}{5}km}{hr}$

iv) If the ratio of the speeds of A and B is a:b, then the ratio of the times taken by them to cover the same distance is $\frac{1}{a}:\frac{1}{b}$ or b:a.

Ex.1 A scooterist covers a certain distance at 36kmph. How many meters does he cover in 2 minutes?

Sol: Speed = 36kmph=36 x(5/18)m/sec=10m/sec

So, Distance covered in 2 min= 10x2x60=1200m

Ex.2. There are two towns A and B. Anil goes from A to B at 40kmph and comes back to the starting point at 60kmph.what is his average speed during the whole journey?

Sol : average speed =$\left(\frac{2xy}{x+y}\right)$kmph

$=\left(\frac{2\times 40\times 60}{40+60}\right)$

= 48kmph

Ex.3. A man travels for 5 hrs 15min. if he covers the first half distance at 60kmph and the rest at 45kmph. Find the total distance travelled by him.

Sol :let the distance be x then

$$\left(\frac{\frac{x}{2}}{60}+\frac{\frac{x}{2}}{45}\right)=\frac{21}{4}\left(\frac{x}{120}+\frac{x}{90}\right)=\frac{21}{4}$$

7x=90x21

X=270m

PROBLEMS ON TRAINS

1. Time taken by a train x metres long in passing a signal post or pole or a standing man = Time taken by the train to cover x metres.

2. Time taken by a train x metres long in passing stationary of length y metres = Time taken by the train to cover (x+y) metres.

3. Suppose two trains or two bodies are moving in the same direction at u kmph and v kmph such that u>v , then their relative speed =(u-v)kmph.

4. If two trains of length x km and y km are moving in the same direction at u kmph and v kmph, where u>v then time taken by faster train to cross the slower train

$$=\left(\frac{x+y}{u-v}\right)hrs$$

5. Suppose two trains or two bodies are moving in the opposite direction at u kmph and v kmph such that u>v , then their relative speed =(u+v)kmph.

6. If two trains of length x km and y km are moving in the opposite direction at u kmph and v kmph, where u>v then time taken by trains to cross the other train.

$$=\left(\frac{x+y}{u+v}\right)hrs$$

7. xkmph= $\left(x \times \frac{5}{18}\right)$m/sec

8. y metres/sec = $\left(y \times \frac{18}{5}\right)$km/hr

Ex.1. Find the time taken by a train 180m long, running at 72 kmph in crossing an electric pole.

Sol :Speed of the train = $\frac{\left(72 \times \frac{5}{18}\right)m}{\text{sec}}$ =20m/s

Distance moved in passing the pole= 180m.

Required time taken = $\frac{180}{20}$ =9 sec.

Ex.2. A train 140m long is running at 60kmph. In how much time will it pass a platform 260m long?

Sol: speed of the train =$\left(60 \times \frac{5}{18}\right)$m/sec

=50/3 m/sec

Distance covered in passing the platform=(140+260)=400m

So, time taken =$\left(400 \times \frac{3}{50}\right) = 24 \text{ sec}$

Ex.3.a man is standing on a railway bridge which is 180m long. He finds that a train crosses the bridge in 20 sec but himself in 8 seconds. Find the length of the train and its speed.

Sol: let the length of the train be x mtrs.

Then the train covers x mtrs in 8 sec and (x+180) in 20seconds.

$\therefore \frac{x}{8} = \frac{x + 180}{20}$

=20x=8(x+180)

=>x=120m

So length of the train= 120m

Speed of the train =120/8 m/s

=15m/sec=15× 18/5

= 54kmph.

Ex.4. A train 220m long is running with a speed of 59kmph. In what time it will pass a man who is running at 7 kmph in the direction opposite to that in which the train is going?

Sol: speed of the train relative to man= 59+7 kmph

$= \frac{\left(66 \times \frac{5}{18}\right) m}{\text{sec}}$

=55/3 m/s

Time taken by the train to cross the man=time taken by it to cover 220 m at 55/3 m/s.

$=\left(220 \times \frac{3}{55}\right) \text{sec}$ = 12 sec.

PROBLEMS ON BOATS AND STREAMS

IMPORTANT POINTS

1. In water the direction along the stream is called downstream. And the direction againstthe stream is called upstream.

2. if speed of a boat in still water is u km/hr and the speed of the stream is vkm/hr then:

Speed downstream=(u+v) km/hr

Speed upstream=(u-v) km/hr

3. if the speed downstream is a km/hr and the speed upstream is b km/hr then:

Speed in still water=(a+b)/2

Rate of stream=(a-b)/2

Ex1. A man can row upstream at 7 kmph and downstream at 10kmph. Find the mans rate in still water and the rate of current.

Sol : rate in still water=(10+7)/2

= 8.5kmph

Rate of current=(10-7)/2

= 1.5kmph

Ex.2. A man rows downstream 27km and upstream 18km, taking 3 hours each time. What is the velocity of the current?

Sol : rate downstream=27/3 = 9kmph

Rate upstream= 18/3 = 6kmph

Velocity of current= (9-6)/2=1.5km/hr

Ex.3. A man can row 12kmph in still water. It takes him twice as long as to row up as to row down the river. Find the rate of stream.

Sol : let mans rate upstream be x kmph.

Then his rate downstream =2x kmph

Rate in still water=(x+2x)/2 = 3x/2 kmph

= 3x/2=12

= x=8kmph

i.e. rate upstream= 8km/hr

rate downstream= 16kmph

hence rate of stream= (16-8)/2 = 4kmph.

Ex.4.A man can row 8kmph in still water and the river is running at 2kmph. If the man takes 1 hour to row to a place and back, how far is the place?

Sol : Man's rate downstream=(8+2) kmph = 10kmph.

Man's rate upstream= (8-2) kmph = 6kmph.

Let the required distance be x km.

Then , $\frac{x}{10} + \frac{x}{6} = 1$

3x+5x=30

x= 3.75km.

hence the required distance is 3.75km.

UNIT - 3

SIMPLE INTEREST

GENERAL CONCEPTS:

PRINCIPAL OR SUM

THE MONEY BORROWED OR LEAD OUT FOR A CERTAIN PERIOD IS CALLED THE PRINCIPLE OR THE SUM.

INTEREST

EXTRA MONEY PAID FOR USING OTHERS MONEY IS CALLED INTEREST.

SIMPLE INTEREST:IF THE INTEREST ON A SUM BORROWED FOR A CERTAIN PERIOD IS RECKONED UNIFORMLY,THEN IT IS CALLED SIMPLE INTEREST.

FORMULAE

LET PRINCIPLE=P,RATE=R% PER ANNUM AND TIME=T YEARS.THEN,

1. S.I.=(P*R*T/100).

2. P=(100*S.I./R*T); R=(100*S.I./P8T) AND T=(100*S.I./P*R)

SAMPLE PROBLEMS:

EX. 1. FIND THE SIMPLE INTEREST ON RS. 68000 AT 16*2/3% PER ANNUM FOR 9 MONTHS?

SOL. P=RS. 68000, R=50/3% P.A AND T=9/12 YEARS=3/4 YEARS.

S.I.=(P*R*T/100)= RS.(68000*50/3*3/4*1/100)= RS. 8500.

EX. 2. AT WHAT RATE PERCENT PER ANNUM WILLA SUM OF MONEY DOUBLE IN 16 YEARS?

SOL. LET PRINCIPLE = P. THEN,S.I = P AND T = 16 YRS.

RATE = (100*P/P*16)%=6*1/4% P.A.

EX. 3.A SUM AT SIMPLE INTEREST AT 13*1/2% PER ANNUM AMOUNTS TO RS.2502.50 AFTER 4 YEARS.IND THE SUM?

SOL. LET SUM BE RS. X. THEN,S.I. = RS.(X*27/2*4*1/100) = RS.27X/50.

AMOUNT = RS.(X+27X50) = RS. 77X/50.

77X/50 = 2502.50 ⇔ X=2502.50*50/77=1625.

HENCE, SUM = RS. 1625.

EX. 4.A CERTAIN SUM O MONEY AMOUNTS TO RS. 1008 IN 2 YEARS AND TO RS. 1164 IN 3*1/2 YEARS. FIND THE SUM AND THE RATE O INTEREST?

SOL. S.I FOR 1 ½ YEARS = RS. (1164-1008) = RS. 156.

S.I. FOR 2 YEARS = RS.(156 *2/3 * 2) = RS.208

PRINCIPAL = RS. (1008 – 208) = RS. 800.

NOW, P= 800, T = 2 & S.I= 208.

RATE = (100 * 208 / 800 * 2) % = 13%.

COMPOUND INTEREST

SOME TIMES IT SO HAPPENS THAT THE BORROWER AND THE LENDER AGREE TO FIX UP A CERTAIN O TIME, SAY YEARLY OR HALF YEARLY AND QUARTERLY TO SETTLE THE PREVIOUS ACCOUNT.

IN SUCH CASES, THE AMONUT AFTER FIRST UNIT OF TIME BECOMES THE PRINCIPLE FOR THE SECOND UNIT, THE AMOUNT AFTER SECOND UNIT BECOMES THE PRINCIPAL FOR THE THIRD UNIT AND SO ON

AFTER THE SPECIFIED PERIOD , THE DIFFERENCE BETWEEN THE AMOUNT AND THE MONEY BORROWED IS CALLED THE COMPOUND INTEREST FOR THAT PERIOD.

IMPORTANT ACTS AND FORMULAE

LET PRINCIPAL = P, RATE = R% PER ANNUM, TIME = N YEARS

I. WHEN INTEREST IS COMPOUND ANNUALLY:

   AMOUNT = $P (1+ R/ 100)^N$

II. WHEN INTEREST IS COMPOUND HALF YEARLY:

   AMOUNT = P (1+ (R/2) / 100)2N

III. WHEN INTEREST IS COMPOUND QUARTELY:

   AMOUNT = P (1+ (R/4) / 100)4N

IV. WHEN INTEREST IS COMPOUND ANNUALLY BUT TIME IS IN FRACTION, SAY 3 2/5 YEARS

   AMOUNT = P (1 + R / 100)3 * (1+ 2/5 R / 100)

V. WHEN RATES ARE DIFFERENT FOR DIFFERENT YEARS, SAY R1%, R2%, R3% FOR 1^{ST}, 2^{ND} AND 3^{RD} YEAR RESP.

   THEN, AMOUNT = P (1+R1/100) (1+R2/100) (1+R3/100)

VI. PRESENT WORTH OF RS. X DUE N YEARS HENCE IS GIVEN BY:

   PERSENT WORTH = X/(1+R/100)N.

SAMPLE PROBLEMS

EX. 1.FIND COMPOUND INTEREST ON RS.7500 AT PER ANNUM FOR 2 YEARS,COMPOUNDED ANNUALLY?

SOL. AMOUNT = RS.[7500*(1+4/100)^2]=RS.[7500*26/25*26/25) = RS 8112.

C.I. = RS.(8112-7500)= RS. 612.

EX. 2.FIND COMPOUND INTEREST ON RS.8000 AT 15% PER ANNUM FOR 2 YEARS 4 MONTHS, COMPOUNDED ANNUALLY?

SOL. TIME = 2 YEARS 4 MONTHS = 2*4/12 YEARS = 2*1/3 YEARS.

AMOUNT = RS.[8000*(1+15/100)^2*(1+1/3*15/100)]

= RS.(8000*23/20*23/20*21/20)

=RS.11109

C.I = RS. (11109-8000) = RS. 3109.

EX. 3.IND THE COMPOUND INTEREST ON RS.16000 AT 20% PER ANNUM FOR 9 MONTHS,COMPOUNDED QUARTERLY?

SOL. PRINCIPAL = RS. 16000; TIME = 9 MONTHS = 3 QUARTERS;

RATE = 20% PER ANNUM = 5% PER QUARTER.

AMOUNT = RS.[16000*(1+5/100)^3]

= RS. (16000*21/20*21/20*21/20)

= RS. 18522

C.I. = RS.(18522-16000)=RS. 2522.

PARTNERSHIP

1. PARTNERSHIP: WHEN TWO OR MORE THAN TWO PERSONS RUN A BUSSINESS JOINTLY, THEY ARE CALLED PARTNERS AND THE DEAL IS KNOWN AS PARTNERSHIP

2. RATIO OF DIVISION OF GAINS

   (i) WHEN INVESTMENT OF ALL THE PARTNERS ARE FOR THE SAM\E TIME, THE GAIN OR LOSS IS DISTRIBUTED AMONG THE PARTNERS IN THE RATIO OF THEIR INVESTMENTS

   SUPPOSE A & B INVEST RS. X & RS.Y RESP. OR A YEAR IN A BUSSINESS, THEN AT THE END OF THE YEAR:

   (A'S SHARE OF PROFIT) : (B'S SHARE OF PROFIT) = X:Y

   (ii) WHEN INVEST ARE DIERENT TIME PERIODS, THEN EQUIVALENT CAPITALS ARE CALCULATED FOR A UNIT O TIME BY TAKING (CAPITAL * NO. OF UNIT OF TIME). NOW, GAIN OR LOSS IS DIVIDED INTO THE RATIO OF THESE CAPITALS.

   SUPPOSE A INVEST RS.X OR P MONTHS AND B INVEST RS.Y FOR Q MONTHS, THEN

   (A'S SHARE OF PROFIT) : (B'S SHARE OF PROFIT) = XP:YQ

3. WORKING AND SLEEPING PARTNERS: A PARTNER WHO MANAGES THE BUSSINESS IS KNOWN AS WORKING PARTNER AND THE ONE WHO SIMPLY INVEST THE MONEY IS KNOWN AS SLEEPING PARTNER.

SAMPLE PROBLEMS:

EX. 1.FOUR MILMEN RENTED A PASTURE. A GRAZED 24 COWS FOR 3 MONTHS;B 10 COWS FOR 5 MONTHS ;C 35 COWS OR 4 MONTHS AND D 21 COWS FOR 3 MONTHS.IF A'S SHARE O RENT IS RS. 720,FIND THE TOTAL RENT OF THE FIELD?

SOL. RATIO O SHARES OF A,B,C,D = (24*3):(10*5):(35*4)(21*3)

=75:50:140:63.

LET TOTAL RENT BE RS. X. THEN,A'S SHARE = RS. 72X/325.

72X/325=720⇔ X=720*325/72=3250.

HENCE,TOTAL RENT OF THE FIELD IS RS.3250.

EX. 2.A INVESTMENT RS 76000 IN A BUSSINESS. ATER FEW MONTHS,B JOINED HIM WITH RS.57000. AT THE END OF THE YEAR, THE TOTAL PROFIT WAS DIVIDED BETWEEN THEM IN THE RATIO 2:1.AFTER HOW MANY MONTHS DID B JOIN?

SOL. SUPPOSE B JOINED AFTER X MONTHS.THEN,B'S MONEY WAS INVESTED FOR(12-X) MONTHS

76000*12/57000*(12-X)=2/1⇔912000=114000(12-X)

⇔114(12-X)=912⇔(12-X) = 8⇔X=4

HENCE,B JOINED AFTER 4 MONTHS.

EX. 3.A,B,C ENTER INTO PARTNERSHIP. A INVESTS 3 TIMES AS MUCH AS B INVESTS AND B INVEST TWO THIRD O WHAT C INVESTS.AT THE END O THE YEAR,THYE PROFIT EARNED IS RS.6600.WHAT IS THE SHARE OF B?

SOL. LET C'S CAPITAL = RS X.THEN,B'S CAPITAL = RS. 2/3*X.

A'S CAPITAL = RS(3*2/3*X)=RS. 2X.

RATIO O THEIR CAPITALS = RS 2X:2/3*X:X=6:2:3.

HENCE,B'S SHARE = RS.(6600*2/11) = RS.1200.

PROFIT OR LOSS

COST PRICE:

THE PRICE AT WHICH AN ARTICLE IS PURCHASED, IS CALLED ITS COST PRICE, ABBRIVATED AS C.P

SELLING PRICE:

THE PRICE AT WHICH AN ARTICLE IS SOLD, IS CALLED ITS SELLING PRICE, ABBRIVATED AS S.P

PROFIT OR GAIN = (S.P) – (C.P)

LOSS = (C.P) – (S.P)

AN IMPORTANT RESULT: LOSS OR GAIN IS RECKONED ON C.P

FORMULAE:

(i) GAIN = (S.P) – (C.P)

(ii) GAIN% = (GAIN * 100/C.P)

(iii) LOSS = (C.P) – (S.P)

(iv) LOSS% = LOSS * 100/C.P)

(v) S.P = (100+ GAIN %) / 100 * C.P

(vi) S.P = (100+ LOSS %) / 100 * C.P

(vii) C.P = (100/(100+GAIN%) * S.P

(viii) C.P = (100/(100+LOSS%) * S.P

SAMPLE PROBLEMS:

EX. 1. IF THE COST PRICE IS 96% OF THE SELLING PRICE,THEN WHAT IS THE PROFIT PERCENT?

SOL. LET S.P. =RS 100.THEN,C.P = RS. 96;PROFIT = RS.4.

PROFIT% N= (4/96*100)%=25/6%=4.17%

EX. 2.FIND COST PRICE., WHEN SELLING PRICE=RS40.60,GAIN=16%

SOL. COST PRICE = RS(100/116*40.60)=RS 35.

EX. 3.A MAN BUYS AN ARTICLE OR RS.27.50 AND SELLES IT FOR RS 28.60.FIND HIS GAIN PERCENT?

SOL. COST PRICE = RS 27.50,SELLING PRICE =RS 28.60.

SO, GAIN = RS.(28.60-27.50) = RS.1.10.

GAIN% = (1.10/27.50*100)%=4%

EX. 4.IF A RATIO IS PURCHASED FOR RS.490 AND SOLD OR RS.465.50,FIND THE LOSS PERCENT?

SOL. COST PRICE = RS 490,SELLING PRICE = RS 465.50.

LOSS = RS.(490-465.50)=RS. 24.50

LOSS% = (24.50/490*100)% = 5%.

UNIT - 4

TRUE DISCOUNT

FORMULA

True discount T.D= Interest on P.W

Amount=P.W+T.D

1. P.W=$\frac{100 \times Amount}{100 + (R \times T)} = \frac{100 \times T.D}{R \times T}$

2. T.D=$\frac{(P.W) \times R \times T}{100} = \frac{Amount \times R \times T}{100 + (R \times T)}$

3. sum=$\frac{(S.I) \times (T.D)}{(S.I) - (T.D)}$

4.(S.I)-(T.D)=S.I on T.D

5. when the sum is put at compound interest, then p.w=$\frac{Amount}{\left(1 + \frac{R}{100}\right)^T}$

SAMPLE PROBLEMS

1. find the present worth of Rs.930 due 3years hence at 8% per annum. Also find the discount.

P.W=$\frac{100 \times Amount}{100 + (R \times T)} = Rs. \frac{100 \times 930}{100 + (8 \times 3)} = Rs. 750.$

2. The true discount on a bill due 9 months hence at 12% per annum is Rs.540 find the amount of the bill and its present worth.

Le amount be Rs. X then,

$$\frac{x \times R \times T}{100 + (R \times T)} = T.D$$

$$\frac{x \times 12 \times \frac{3}{4}}{100 + \left(12 \times \frac{3}{4}\right)} = 540$$

$$x = \left[\frac{540 \times 109}{9}\right] = Rs. 6540$$

∴ Amount=RS. 6540

P.W= Rs. (6540-540)=RS.6000

3.The true discount on a certain sum of one due 3years hence is Rs. 250 and the simple interest on the same sum for th same time and at the same rate is Rs.375.

T.D=Rs.250 and S.I= Rs. 375

Sum due= $\frac{S.I \times T.D}{(S.I) - (T.D)}$

$=[(375 \times 250)/(375 - 250)$

=Rs.750

Rate= $\frac{100 \times 375}{750 \times 3}\% = 16\frac{2}{3}\%$

4.The different between the simple interest and true discount on a certain sum of money for 6 month at $12\frac{1}{2}\%$ per annum is Rs. 25. Find the sum

Let the sum be Rs.x, then,

T.D= $\frac{x \times \frac{25}{2} \times \frac{1}{2}}{100 + \left(\frac{25}{2} \times \frac{1}{2}\right)} = x \times \frac{25}{4} \times \frac{4}{425} = \frac{x}{7}$

S.I $= \left[x \times \frac{25}{2} \times \frac{1}{2} \times \frac{1}{100}\right] = \frac{x}{16}$

$\frac{x}{.16} - \frac{x}{17} = 25$

17x-16x=25×16×17

X=6800

Hence, sum due=Rs. 6800.

5.A bill falls due in 1 year. The creditor agrees to accept immediate payment of the half and to defer the payment of the other half for 2 years. By this arrangement he gains Rs. 40 what is the amount of the bill, if the money be worth $12\frac{1}{2}\%$?

Let the sum be Rs. X then,

$$\left[\frac{x}{2} + \frac{\frac{x}{y} \times 100}{100 + \left(\frac{25}{2} \times 2\right)}\right] - \frac{x \times 100}{100 + \left(\frac{25}{2} \times 1\right)} = 40$$

$= \frac{x}{2} + \frac{2x}{5} - \frac{8x}{9}$ **=40**

X=3600

Amount of the bill=Rs. 3600

BANKERS DISCOUNT

Formula:

Banker's Gain: (B.G)=(B.D)-(T.D)

1. B.D=S.I on bill for unexpired time.
2. B.G=(B.D)-(T.D)=S.I on T.D=$\frac{(T.D)^2}{P.W}$.
3. T.D=$\sqrt{P.W \times B.G.}$
4. B.D=$\left(\frac{Amount \times Rate \times Time}{100}\right)$
5. T.D=$\left[\frac{Amount \times Rate \times Time}{100 + (Rate \times Time)}\right]$
6. Amount=$\left(\frac{B.D \times T.D}{B.D - T.D}\right)$
7. T.D=$\left(\frac{B.G \times 100}{rate \times Time}\right)$

PROBLEMS:

1.if the true discount on a certain sum due 6 months hence at 15% is Rs.120 what is the banker's discount on the same sum for the same time and at the same rate?

B.G=S.I on T.D=Rs.$120 \times 15 \times \frac{1}{2} \times \frac{1}{100}$

(B.D)-(T.D)=Rs. 9

B.D=RS.(120+9)=Rs.=129

2. The banker's discount on Rs.1800 at 12% per annum is equal to the true discount on Rs. 1872 for the same time at the same rate. Find the time.

S.I on Rs.1800=T.D on Rs.1872

P.W of Rs.!872 is Rs.1800

Rs.72 is S.I on Rs.1800 at 12%

Time=$\left[\frac{100 \times 72}{12 \times 1800}\right]$years=1/3year=4 month

3.The banker's discount and the true discount on a sum of money due 8 months hence are Rs. 120 and Rs. 110 respectively. Find the sum and the rate period.

Sum=$\left[\frac{B.D \times T.D}{B.D - T.D}\right]$=$\left[\frac{120 \times 110}{120 - 110}\right]$=1320.Rs

Since B.D is S.I on the sum due, so S.I on Rs. 1320 for 8 months is Rs.120

Rate=$\left[\frac{100 \times 120}{1320 \times \frac{2}{3}}\right]$%=$13\frac{7}{11}\%$

4.The present worth of a bill due sometime hence is Rs. 1100 and the true discount on the bill is Rs. 110. Find the banker's discount and the banker's gain.

T.D=$\sqrt{P.W \times B.G.}$

B.G=$\frac{(T.D)^2}{P.w}$=Rs.$\left[\frac{110 \times 110}{1100}\right]$=Rs.11

B.D=(T.D+B.G)=RS.(110+11)=Rs.121

5. What rate percent does a man get for his money when in discounting a bill due 10 month hence, he deducts 10% of the amount of the bill?

Let, amount of the bill=Rs.100. money deducted=Rs.10

Money received by the holder of the bill=Rs.(100-10)=Rs. 90

S.I on Rs.90 for 10 months=Rs.10.

Rate=$\left[\frac{100 \times \frac{10}{10}}{90 \times \frac{10}{12}}\right]$%=$13\frac{1}{4}\%$.

PERCENTAGE

Formulae

1. Concept of percentage: By a certain **percent**, we mean that many hundredths. Thus, x percent means x hundredths, written as x%

   To express x% as a fraction: we have, $x\% = \frac{x}{100}$

   To express x% as a percent: we have, $\frac{a}{b} = \left(\frac{a}{b} \times 100\right)\%$

2. Result on population: Let the population of a town be p now and suppose increases at the rate of R% per annum, then:

- Population after n years $= p\left(1 + \frac{R}{100}\right)^n$
- Population n years ago $= \frac{P}{\left(1 + \frac{R}{100}\right)n}$

3. Result on Depreciation: Let the present value f a machine be P. suppose it depreciates at the rate of R% per annum

   1. Value of the machine after n years $= P\left(1 - \frac{R}{100}\right)^n$
   2. Value of the machine n years ago $= \frac{P}{\left(1 - \frac{R}{100}\right)^n}$

4. If A is R% more than B, then B is less than A by $\left[\frac{R}{(100 + R)} \times 100\right]\%$

If A is R% less than B, then B is more than A by $\left[\frac{R}{(100 - R)} \times 100\right]\%$

Problems:

1.**Express each of the following as a decimal:**

$6\% = \frac{6}{100} = 0.06$

$0.2\% = \frac{0.2}{100} = 0.002$

2.(a) 2 is what precent of 50?

(b)1/2is what percent of 1/3?

(c)what precent of 7 is 84?

(d)what precent of 2 metric tonnes is 40 quintals?

(a) Required percentage $= \left[\frac{2}{50} \times 100\right]\% = 4\%$

(b) Required percentage $\left[\frac{1}{2} \times \frac{3}{1} \times 100\right]\% = 150\%$

(c) Required percentage $\left[\frac{84}{7} \times 100\right]\% = 1200\%$

(d)1 metric tonne= 10 quintals.

∴ Required percentage $= \left[\frac{40}{20} \times 100\right]\% = 200\%$

3.Find the missing figures:

(a)?% of 25=2.25

Let x% of 25=2.125. then, $\frac{x}{100} \times 25 = 2.125 \leftrightarrow x = (2.125 \times 4) = 85$

(b)9% of ?=63.

Let 9% of x=6.3. then, $\frac{9}{100}$x=6.3 $\leftrightarrow x = \left(6.3 \times \frac{100}{9}\right)$=70

4.sixty-five percent of a number is 21 less than four-fifth of that number what is the number?let the number be x

Then, $\frac{4}{5}$x-(65% of x)=21 $\leftrightarrow \frac{4}{5}$x-$\frac{65}{100}$x=21 $\leftrightarrow 15x = 2100 \leftrightarrow x = 140.$

5. **If 50% of(x-y)=30% of (x=y), then what percent of x is y?**

50% of (x-y)=30% of (x+y) $\leftrightarrow \frac{50}{100}$(x-y)=$\frac{30}{100(x+y)}$

5(x-y)=3(x+y) ↔ 2x=8y↔ x=4y.

∴ Required percentage= [y/x ×100]%=[$y/4y$ ×100]% = 25%

RATIO & PROPORTION

Ratio:

The ratio of two quantities a and b in the same units, is the fraction a/b and we write it as a:b

RULE: the multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Proportion: The equality of two ratios is called proportion.

If a:b=c:d, we write, a:b::c:d and we say that a,b,c,d are in proportion

Here a and d are called extremes, while b and c are called mean terms.

PRODUCT OF MEANS= PRODUC OF EXTREMES.

Thus, a:b::c:d↔ (bxc)=(axd)

Mean proportional: mean proportional between a and b is $\sqrt{ab}$

Comparison of Ratios: we say that(a:b)>(c:d) $\leftrightarrow \frac{a}{b} > \frac{c}{d}$

Compounded Ratios: the compounded ratio of the ratios(a:b),(c:d),(e:f)is (ace:bdf)

VARIATION:

(a) We say that x is directly proportional to y, if x=ky for some constant k and we write ,x$\propto y$

(b)we say that x is inversely proportional to y, if xy=k for some constant k and we write ,x$\propto \frac{1}{y}$

Problems

1. **If a:b=5:9 and b:c=4:7, find a:b:c.**

a:b=5:9 and b:c=4:7=[4x9/7]:[7x9/4]=9:63/4

a:b:c=5:9:63/4=20:36:63

2.**if x:y=3:4, find (4x+5Y):(5x-2y).**

x/y=3/4 $\leftrightarrow \frac{4x+5y}{5x-2y} = \frac{4\left[\frac{x}{y}\right]+5}{5\left[\frac{x}{y}\right]-2} = \frac{4\times\frac{3}{4}+5}{5\times\frac{3}{4}-2} = \frac{(3+5)}{\left(\frac{7}{4}\right)} = \frac{32}{7}$

3.divide Rs.1162 among A,B,C in the ratio 35:28:20?

Sum of ratio terms=(35+28=20)=83

A's share=Rs.[1162×35/83]=Rs.490

B's share=Rs.[1162×28/83]=RS.392

C's share=RS. [1162×20/83]=Rs.280

4.a bag contains 50p,25p and 10p coins in the ratio 5:9:4, amonunting to Rs. 205. Find the number of coins of each type.

Let the number of50p, 25p and 10p coins be 5x,9x and 4x respectively.

Then, 5x/2+9x/4+4x/10= 206

50x+45x+8x=4120↔ 103x=4120↔ $x = 40.$

∴ Number of 50p coins=5×40=200

Number of 25p coins=9×40=360

Number of 10p coins=4×40=160

5. A mixture contains alcohol and water in the ratio 4:3. If 5 litres of water is added to the mixture, the ratio become 4;5. Find the quantity of alcohol in the given mixture.

let the quantity of alcohol and water be 4x litres and 3x litres respectively.
Then,4x/3x+5=4/5↔ $20x = 4(3x + 5) \leftrightarrow 8x = 20$

x=2.5

∴ Quantity of alcohol= (4x2.5)litres=10 litres.

UNIT - 5

ALGEBRA

ALGEBRIC EXPRESSIONS & INEQUALITIES.

RESULTS:

1. Quantities are said to be in Arithmetic progression when they increase or decrease by a constant difference to get the next or the previous term respectively.

An AP cane be represented by a, a+d, a+2d, ….., a+(n-1)d.

where a is the first term, d is the common difference.

The nth term of the A.P. is denoted by

$T_n = a+(n-1)d$

The sum of n terms of an A.P. is

$S_n = n[2a+(n-1)d]/2$

also, $S_n = n(\text{First term} + \text{Last term})/2$

Examples:

1) Find the 12^{th} term of an A.P. whose first term is 2 and common difference is 3.

Sol: Here a=2, d= 3 and n=12

so, $T_{12} = 2+(12-1)x3$

$= 2+33= 35.$

2) Find the number of terms in an A.P. if the first term is 3, common difference is 2 and the last term is 27.

Sol: Given a=3, d=2**,** $T_n=27$

$T_n = a+(n-1)d$

$27=3+(n-1)2$

$n-1 =12$

$n= 13.$

so, there are 13 terms in the A.P.

3) Find the first term and the common difference of an A.P. If the 3rd term is 4 and the 9th term is 13.

Sol: If the first term, common difference, and no. of terms are a,d and n respt. then from problem,

$a+2d = 4$……(1)

$a+8d=13$……(2)

By subtracting the first equation, from the second equation, we get

$d=3/2$

substituting the value of d in any equation we get

$a=1$

so first term is 1 and the common difference is 3/2.

4) Find the sum to 17 terms of an A.P. whose first term is 4 and common difference 3/4.

Sol: Sum to 17 terms is

$S_{17} = 17[2x4 + (17-1)x3/4]/2$

$= 17x(20/2)$

$= 17x10=170$

5) Find the arithmetic mean of the A.P. with 29 terms whose first term is 1 and common difference is 1.5.

Sol: First term =1

Last term = $a+(n-1)d$

$=1+(29-1)x1.5$

$= 1+42 = 43$

Arithmetic mean = (First term + Last term)/2

$= (1+43)/2= 44/2 = 22.$

GEOMETRIC PROGRESSION

Quantities are said to be Geometric progression when the ratio of any quantity to the quantity that follows is the same.

Any term of a GP can be obtained by multiplying the previous term by a constant factor called the common ratio 'r'.

A G.P. can be represented by a, ar, ar^2,….. ar^{n-1}.

The nth term of the G.P. is ar^{n-1}.

Sum of first n terms:

if r<1, then

$S_n = a(1-r^n)/(1-r)$

If r>1, then

$S_n = a(r^n -1)/(r-1)$

Examples:

1) Find the 5^{th} term of the G.P. whose first term is 3 and common ratio is 1/3.

Sol: Given, a=3, r =1/3

5^{th} term = $a.r^4$

$= 3.(1/3)^4 = 1/27.$

2) Find the sum to 4 terms of a G.P. whose first term is 2 and common ratio is 2/3.

Sol: Here r=2/3<1, then

$S_n= a(1-r^n)/(1-r)$

$S_4=2[1-(2/3)^4]/\{1-(2/3)\}$

$= 2(65/81)/(1/3) = 130/27.$

3) Find the number of terms in the G.P. whose first term is 2, sum is (2062/625) and the common ratio is 2/5.

Sol: Given a=2, r = 2/5, and

S_n= 2062/625

- ⇨ $a(1-r^n)/(1-r) = 2062/625$
- ⇨ $2[1-(2^n/5^n)]/(1-2/5) = 2062/625.$
- ⇨ $1-(2^n/5^n) = 2062x3/625x10$
- ⇨ $(2/5)^n = 64/(5^4.5.2) = (2/5)^5$

  Hence, n=5.

4) Find the common ratio of the G.P. whose first and last terms are 3 and 1/729 respectively and the sum of G.P. is (3280/729).

Sol: Since first term>last term,

r<1

Now, sum =[first term-r(last term)]/(1-r)

Hence, (3280/729) = [3-r(1/729)]/1-r

On simplification we get,

r=1/3.

5) Find three numbers in G.P. whose sum is 19 and whose product is 216.

Sol: Let the three numbers be a/r, a, ar.

Given that

a/r.a.ar = 216

- ⇨ $a^3 = 216$
- ⇨ a = 6

a/r + a + ar =19

$6r^2-13r + 6= 0$

r =3/2 or r = 2/3

Hence the numbers are 4,6,9.

BINOMIAL EXPANSIONS:

The expression (x+a) is called a binomial .

Binomial expansion for any index:

When n is not a positive integer, the Binomial Expansion is:

$(x+a)^n = x^n + n.x^{n-1}.a + [n(n-1)/2!].x^{n-2}.a^2 + [n(n-1)(n-2)/3!]x^{n-3}.a^3 + ...$

Examples:

1) Find the 5th term in $(a + 2x^3)^{17}$.

Sol: The 5^{th} term of the binomial expression is:

$= {}^{17}C_4 .a^{17-4}.(2x^3)^4$

$= (17x16x15x14/1x2x3x4).16a^{13}.x^{12}$

$= 38080\ a^{13}x^{12}$.

2) Find $(126)^{1/3}$.

Sol: $(126)^{1/3} = (5^3 + 1)^{1/3} = 5(1 + 1/5^3)^{1/3}$

$= 5\{1 + (1/3).(1/5)^3 - (1/9).(1/5)^6 + (1/81).(1/5)^9$.....

$= 5.01329$

3) Find the coefficient of x^{16} in the expansion $(x^2-2x)^{10}$.

Sol: $(x^2 + 2x)^{10} = x^{20}[1-(2/x)]^{10}$.

Coefficient of $x^{16} = {}^{10}C_4.(-2)^4 = 3360$.

4) Find the term independent of x in $[(3/2)x^2 - (1/3x)]^9$.

Sol: General term $= {}^9C_r\ (3x^2/2)^{9-r}.(-1/3x)^r$.

$= (-1)^r\ {}^9C_r\ (3/2)^{9-r}(1/3)^r.x^{19-2r}1/x$

$= 18-3r = 0$

⇨ $r = 6$

Hence, 7^{th} term is independent of x.

5) Find the expansion of $(x^2 + 2x - 1)^3$.

Sol: Consider (2x-1)as a single term, we get

$$(x^2)^3 + 3(x^2)^2(2x-1) + 3x^2(2x-1)^2 + (2x-1)^3$$

$$= x^6 + 6x^5 + 9x^4 - 4x^3 - 9x^2 + 6x - 1.$$

GEOMETRY

AREA

FORMULAE:

I. 1. Area of a rectangle=(length × breadth)

2. Perimeter of a rectangle=2(length + breadth)

II. Area of square= $(\text{side})^2 = \frac{1}{2}(\text{diagonal})^2$

III. Area of four walls of a room = 2(length+breadth) x height.

IV. 1. Area of triangle= $\frac{1}{2}$ x base x height

2. Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$,
where s=$1/2(a+b+c$) and a,b,c are sides of a triangle.

3. Area of an equilateral triangle= $\frac{\sqrt{3}}{4}$ x $(\text{side})^2$.

4. Radius of circumference of an equilateral triangle of side a=$\frac{a}{\sqrt{3}}$.

V. 1. Area of a parallelogram = base x height.

2. Area of a rhombus= $\frac{1}{2}$x(product of diagonals)

3. Area of a trapezium= $\frac{1}{2}$x(sum of parallel sides) x (distance between them)

VI. 1. Area of a circle = πR^2.

2. Circumference of a circle = $2\pi R$

3. Length of an arc = $\frac{2\pi R\theta}{360}$

4. Area of a sector = $\frac{2\pi R^2\theta}{360}$.

SAMPLE PROBLEMS

Ex.1. One side of a rectangular field is 15m and one of its diagonals is 17m. Find the area of the field..

Sol: other side = $\sqrt{17^2 - 15^2}$ =$\sqrt{289 - 225}$= $\sqrt{64}$= 8m

So, area = (15 x 8)m^2 =120 m^2

Ex.2 . Find the area of a square, one of whose diagonals is 3.8 m long.

Sol : Area of the square = $\frac{1}{2}$x (diagonal)^2

= $\frac{1}{2}$x 3.8 x 3.8 = 7.22 m^2

Ex.3. Find the area of the triangle whose sides measure 13cm, 14cm,and 15cm.

Sol : Let a=13cm, b=14cm, and c=15cm.

Then , s=$\frac{1}{2}(a + b + c)$=21cm

So, area = $\sqrt{s(s-a)(s-b)(s-c)}$,

= $\sqrt{21(21-13)(21-14)(21-15)}$

=$\sqrt{21 \times 8 \times 7 \times 6}$ = 84 m^2

Ex.4. Find the area of an equilateral triangle each of whose sides is 8cm.

Sol : Area of the triangle= $\left(\frac{\sqrt{3}}{4} \times 8 \times 8\right)$= $16\sqrt{3}$

VOLUME AND SURFACE AREA

1. CUBOID
   i. **Volume** = (length x breadth x height) cubic units
   ii. **Surface area** = 2(lb+bh+hl) square units.
   iii. **Diagonal** = $\sqrt{l^2 + b^2 + h^2}$ units.
2. CUBE
   i. **Volume** = (side)^3 cubic units
   ii. **Surface area** = 6a^2 square units.
   iii. **Diagonal** = $\sqrt{3}a$ units.
3. **CYLINDER**
   i. **Volume** = ($\pi r^2 h$) cubic units
   ii. **Curved Surface area** = $2\pi rh$ square units.
   iii. **Total surface area**= ($2\pi rh + 2\pi r^2$) square units.
4. **CONE**
   i. **Volume** = ($\frac{1}{3}\pi r^2 h$) cubic units.
   ii. **Curved Surface area** = πrl square units.
   iii. **Total surface area**= ($\pi rl + \pi r^2$) square units.
   iv. **Slant height, l=** $\sqrt{l^2 + r^2}$ units.
5. **SPHERE**
   i. **Volume** = ($\frac{4}{3}\pi r^3$) cubic units.
   ii. **Surface area** = $4\pi r^2$ square units.
6. **HEMI-SPHERE**
   i. **Volume** = ($\frac{2}{3}\pi r^3$) cubic units.
   ii. **Surface area** = $2\pi r^2$ square unitsType equation here.

iii. **Total surface area =** $3\pi r^2$ square units.

SAMPLE PROBLEMS

Ex.1.Find the volume and surface area of a cuboid 16m long, 14 m broad and 7m high.

Sol : Volume = (16 x 14 x 7) m^3 = 1568 m^3

Surface area = 2(16 x 14 + 14 x 7 + 16 x 7) m^2

= 868 m^2

Ex.2. Find the length of longest pole that can be placed in a room 12m long, 8m broad and 9m high.

Sol: Length of the longest pole= length of the diagonal of the room

$= \sqrt{12^2 + 8^2 + 9^2} = \sqrt{289}$ = 17m.

Ex.3. The surface area of a cube is 486cm^2. Find its volume .

Sol: here $6a^2 = 468$

=> $a^2 = 81$ => a= 9cm

Volume = 9 x 9 x9 = 729 cm^2 .

Ex.4. Find the volume and surface area of a sphere of radius 10.5cm.

Sol: Volume = $\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}$

= 4851 cm^3

Surface area = $4\pi r^2 = 4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}$ = 1386 cm^2